# Guided Inquiry 2.6: Derivatives of Inverse Functions

## Overview

The goal of this section is to develop derivative rules for the functions y = ln(x), y = arcsin(x), and y = arctan(x). These are very important functions in their own rights, and they are all defined as **inverses** of existing functions. The fact that they are inverse functions provides a one-size-fits-all method for determining their derivatives. We will briefly review the basic precalculus concepts about inverse functions and then see how the definition of an inverse function – in addition to the Chain Rule – leads to these three derivative rules as well as a general rule for differentiating *any* inverse function.

## Learning objectives

### BASIC learning objectives

Each student will be responsible for learning and demonstrating proficiency in the following objectives PRIOR to the class meeting.

* (*Review*) State the basic facts about inverse functions given on page 121.
* (*Review*) State the definition of the natural logarithm, arcsine and arctangent functions.
* (*Review*) Calculate basic values of the natural logarithm, arcsine, and arctangent functions without a calculator. (For example, ln(e5) and arcsin(½).)
* State the derivative of the natural logarithm function.
* State the derivative of the arcsine function.

**Reminder**: [Wolfram|Alpha](http://www.wolframalpha.com) gives you the ability to practice differentiation rules as much as you want. Make up a function to differentiate, take its derivative by hand, then check with W|A.

### ADVANCED learning objectives

The following objectives should be mastered by each student DURING and FOLLOWING the class session through active work and practice:

* Mimic the derivative of the arcsine function derivative to build a differentiation rule for the arctangent function.
* Differentiate functions involving the natural logarithm, arcsine and arctangent functions.
* Solve problems in context that involve the natural logarithm, arcsine and arctangent functions.
* If g is the inverse of a differentiable function f, give a general formula for the derivative of g in terms of the derivative of f.

## Resources

*Reading*: **Read Section 2.6 in Active Calculus**. Please note, here are the main points to focus on during your reading:

* The formula for the derivative of y = ln(x) and its derivation just above where the formula is stated
* Activity 2.6.2 which uses the formula for the derivative of y = ln(x) with other rules we learned earlier
* The formula for the derivative of y = arcsin(x) and its derivation — this involves some trigonometry obviously, so mark any trig questions you have that arise as you read.

*Viewing*: Watch the following videos at the MTH 201 YouTube Playlist, which have a combined running time of 14 minutes, 24 seconds:

* [Quick Review: Derivatives of inverse functions](http://www.youtube.com/watch?v=chdkxtt8XQo&list=PL9bIjQJDwfGuXQHuS5Jkmum_CFILoCZX-&index=46) (3:25)
* [Examples of derivatives with the natural logarithm](http://www.youtube.com/watch?v=jhBhSerqbyU&list=PL9bIjQJDwfGuXQHuS5Jkmum_CFILoCZX-&index=47) (6:30)
* [Derivatives involving arcsin(x)](http://www.youtube.com/watch?v=pEEQNdttZsw&list=PL9bIjQJDwfGuXQHuS5Jkmum_CFILoCZX-&index=48) (5:27)

The following videos are optional but **strongly recommended** if you need a refresher on inverse functions or logarithms. In class we will assume fluency in working with these concepts.

* [Inverse functions, part 1](http://www.youtube.com/watch?v=tf-R8T2oyr4&list=PL476C3C1676343E03&index=18) (5:02)
* [Inverse functions, part 2](http://www.youtube.com/watch?v=KUmWVQc999g&list=PL476C3C1676343E03&index=17) (8:45)
* [Introduction to logarithms](http://www.youtube.com/watch?v=hWw_YQ21xU8&list=PL476C3C1676343E03&index=16) (9:41)
* [Using logarithms to solve exponentials](http://www.youtube.com/watch?v=YbSOUE2Xdpw&list=PL476C3C1676343E03&index=14) (12:07)

More such videos can be found at the [MTH 110 (Intermediate Algebra) playlist](http://www.youtube.com/playlist?list=PL476C3C1676343E03).

## Exercises

The exercises are found on the Google Form linked below.

## Turn-in instructions

Go to the web form located at the following link and type in your answers: <http://bit.ly/1902ilg>